



Subject Name: PROBABILITY THEORY AND STOCHASTIC PROCESSE

Prepared by (Faculty (s) Name): Mrs. K.VANISREE (Assoc.prof) Mrs.Y.JALAJAKSHI (Asst.Prof)

Year and Sem, Department: II Year I Sem ECE Dept.

Important points / Definitions: (Minimum 15 to 20 points covering complete topics in that unit)

1. P(A)+P(A-bar)=1. Or P(A-bar)=1-P(A)

2. Since A union A-bar = S, P(A union A-bar)=1

3. The probability of the impossible event is 0, i.e P(O)=0

4. If A subset B, then P(A) <= P(B).

5. If A and B are two incompatible events, and therefore, P(A-B)=P(A)-P(A intersection B).and P(B-A)=P(B)-P(A intersection B).

6. P(B | A) = P(A intersection B) / P(A), P(A) != 0

7. P(A intersection B) = P(A)P(B | A) = P(B)P(A | B)

8. P(A intersection B) = P(A)P(B)

9. F(x) = P(X <= x)

10. f(x) = d/dx F(x)

11. P_X(k) = (e^-lambda * lambda^k) / k!

p_X(k) = nCk * p^k * (1-p)^(n-k) k = 0, 1, ..., n

12.

f_X(x) = 1 / (sqrt(2*pi)*sigma_X) * e^(-1/2 * ((x-mu_X)/sigma_X)^2), f_X(x) = 1 / (sqrt(2*pi)) * e^(-1/2 * x^2)

13.

F_X(x | B) = P[{X <= x} | B] = P[{X <= x} intersection B] / P(B) P(B) != 0

14.



15.
$$f_x(x/B) = \frac{d}{dx} F_x(x/B)$$

Short Questions (minimum 10 previous JNTUH Questions – Year to be mentioned)

Unit-I: Probability & Random Variable

SHORT ANSWER QUESTIONS:-

1. What are the Axioms of Probability? [Dec- 2014],[Mar-2017]
2. Explain the types of Random Variables. [Dec- 2014]
3. What are the properties of Probability Distribution function? [Dec- 2014]
4. Define a Random Variable. [Nov-2015],[Mar-2017]
5. State theorem of probability [Mar-2017]
6. Distinguish between Binomial and Poisson distribution. [May-2017]
7. Write the probability of Probability Density Function. [Nov/Dec 2016]
8. A discrete Random Variable can be defined on a continuous sample space. State whether it is true or false . Give an example to support your claim. [Nov/Dec 2016]
9. State the conditions for which two events are said to be independent. [Nov/Dec 2016]
10. Write two properties of Joint Distribution function of random variables. [Mar 2017]
11. Distinguish between Joint and condition probability [Mar 2017]
12. Discuss Exponential distribution [Mar 2017]
13. What is total probability and Baye’s theorem [Nov-2015]
14. Explain probability density function with example [Nov-2015]
15. Mention the difference between continuous and mixed random variable [Mar- 2017]
16. Write about the Rayleigh density and distribution function [Mar- 2017]
17. Write the conditions for a function to be a random variable. [Apr/May- 2018]
18. Explain the significance of mathematical model of experiments [Apr/May- 2018]

LONG ANSWER QUESTIONS:-

1. State and prove baye’s theorem of probability. [Nov-2010][Dec-2011],[May-2011][Mar- 2017]
2. Let A1,A2,A3 are three mutually exclusive and exhaustive events associated with experiment E1. B1, B2,B3 are three mutually exclusive events associated with experiment with E2.



	B1	B2	B3	B4
A1	3/36	*	5/36	*
A2	5/36	4/36	5/36	14/36
A3	*	6/36	*	*
P(Bj)	12/36	14/36	*	*

i. Find missing probabilities

ii. $P(B3/A1)$ and $P(A1/B3)$

iii. A1, B3 are independent or not?

[Mar-2017]

3. The six sides of a fair die are numbered from 1 to 6. The die is rolled 4 times. How many sequences of the four resulting numbers are possible?

b. Define cumulative distribution function of a random variable.

c. State and prove axioms of probability.

[Mar-2017]

4. In a hostel 60% of the students read Hindi newspaper and 40% read English newspaper and 20% read both Hindi and English newspapers. A student is selected at random. Find the probability that he reads neither Hindi nor English newspapers.

If he reads Hindi newspapers find the probability that he reads English newspaper. [Mar-2017]

5. Define the following and give example for each of the following

i. Discrete and continuous sample space ii. Mutually Exclusive event

[Mar-2017]

6. Two cards are drawn from a deck of 52-cards deck (the first is not replaced)

i. given the first card is a queen, what is the probability that a second card is also queen

ii. given the first card is a queen, what is the probability that a second card is 7

[Mar-2017]

7. Define and explain the following with an example

i. Mutually exclusive events ii. Exhaustive events iii. Equally likely events [Nov-2015] [Mar-

2017]

8. A class room contains 9 boys and 3 girls

i. In how many ways can the teacher choose a committee of '4'?

ii. How many of them will contain at least one girl?

iii. How many of them will contain exactly one girl?

[Nov-2015]

9. State the condition for a function to be a random variable

[Nov-2015]

10. In an experiment where the pointer on a wheel of chance of paun. The possible outcomes are the numbers from 0 to 12 marked on the wheel. The sample space consists of the numbers in the set $\{0 < S \leq 12\}$ and if the random variable 'X' is defined as $X = X(S) = 'S'$ map the elements of the random variable on the real line and explain.

[Nov-2015]

11. Write a short note on i) Poisson distribution ii) Binomial distribution [Nov -2015]

12. Define probability, set and sample spaces

[Nov/Dec-2016]

13. Discuss the relative frequency approach and axiomatic approach of probability.

14. In a box there are 100 resistors whose resistances and tolerances are as shown in the table below. Let A be the event of drawing a 47Ω resistor, B be the event of drawing a resistor with 5% tolerance, and C be the event of drawing a 100Ω resistor. Find $P(A/B)$, $P(A/C)$ and $P(B/C)$.



Resistance(Ω)	Tolerance		Total
	5%	10%	
22	10	14	24
47	28	16	44
100	24	8	32
Total	62	38	100

15. Write the classical and axiomatic definitions of Probability and for a three digit decimal number chosen at random, find the probability that exactly K digits are greater than and equal to 5, for $0 < K < 3$

[Mar-2017]

16. A missile can be accidentally launched if two relays A and B both have failed. The probabilities of A and B failing are known to be 0.01 and 0.03, respectively. It is also known that B is more likely to fail (probability 0.06), if A has failed.

i. What is the probability of an accidental missile launch?

ii. What is the probability that A will fail, if B has failed?

iii. Are the events "A fails" and "B fails" statistically independent? [Nov/Dec-2011].

17. Consider the experiment of tossing two dice simultaneously. If X denotes the sum of two faces, find the probability for $X \leq 6$.

i. A fair coin is tossed 4 times. Find the probability for the longest string of heads appearing to be three as a result of the above experiment.

18. In certain college, 25% of the boys and 10% of the girls are studying

Mathematics. The girls constitute 60% of the student body. If a student is selected at random and studying mathematics, determine the probability that the student is a girl [Nov/Dec-2011].

19. Coin A has a probability of head = $1/4$ and coin B is a fair coin. Each coin is flipped four times. If X is the number of heads resulting from coin A and Y denotes the same from coin B, what is the probability for $X=Y$? [Nov/Dec-2011].

20. A dice is thrown 6 times. Find the probability that a face 3 will occur at least two times.

21. Define probability, set and sample spaces [Mar-2017]

Fill in the Blanks / Choose the Best: (Minimum 10 to 15 with Answers)

1. The probability of any event bounded between

[ANS :0 and 1]



2. A bag contains 3 red balls, 4 white balls and 7 black balls. what is the probability of ball becomes red (or) black ball **ANS :**

5/7]

3. The probability of an event B, given as A event is $p(B/A) = [\text{ANS: } P(B \cap A) / p(A) , p(A) \neq 0]$

4. If two events are said to be mutually exclusive then **[ANS: $A \cap B = \emptyset , p(A \cap C) = 0$]**

5. If two events are occurring simultaneously then the two events are **[ANS: NOT MUTUALLY EXCLUSIVE]**

6. Let the favourable event be 'G' then how to calculate the proof of occurrence of unfavourable

Event **[ANS: $P(G) = 1 - p(G)$]**

7. Given event B a subset of event A then the $P(A/B) = [\text{ANS: } P(A/B) = 1]$

8. Express the total probability by using conditional probability **[ANS: $\sum P(A) = p(A/B_n) \cdot B_n$]**

9. If A & B are two independent events, then the conditional probability **[ANS: $p(A/B) = p(A), p(B/A) = p(B)$]**

10. The probability density function (pdf) is defined by using probability distribution function. **[ANS: DERIVATIVE or $f_x(x) = dF_x(x)/dx$]**

11. When two dice thrown, find the probability of event $A = \{\text{sum} = 7\}$ using axiom 3 **[A]**

a) 1/6 b) 2/36 c) 1/12 d) None

12. For any discrete random variable the cumulative distribution function plot is a **[B]**

a) unit step function b) stair case function c) a & b d) impulse function

13. $f(x) = kx(1-x)$ in $0 < x < 1$ the pdf of k = **[B]**

a) 4 b) 6 c) 7 d) 5

14. Gaussian density function plot in the form of **[D]**

a) Step b) Impulse c) Stair-case d) Ball shaped

15. The cumulative distribution function is defined **[A]**

a) $F_x(x) = p\{X \leq x\}$ b) $F_x(x) = p\{X > x\}$ c) $F_x(x) = p\{X = x\}$ d) NONE

16. $F_x(-\infty/Y) = \underline{\hspace{2cm}}$ and $F_x(\infty/Y) = \underline{\hspace{2cm}}$ **[B]**

a) 0 and 0 b) 0 and 1 c) 1 and 0 d) 1 and 1



17.If the mean of the Poisson variate X is one then $P(X=1)$ [C

]

a) e^{-2} b) e^{-3} c) e^{-1} d) e^0

18.skew is 20, variance is 16,the co-efficient of skew ness value [A

]

a)4/16 b)5/16 c)6/16 d)7/16

19.if $m_1 = 2, E[x^2] = 4$, what is variance and standard derivation [C

]

a)0&1 b)1&0 c)0&0 d)1&1

Unit-II: Operations On Single & Multiple Random Variables

Important points / Definitions: (Minimum 15 to 20 points covering complete topics in that unit)

1.
$$\sigma_x^2 = E(X - \mu_x)^2 = \int_{-\infty}^{\infty} (x - \mu_x)^2 f_x(x) dx$$

2.

$$\sigma_x^2 = \sum_{i=1}^N (x_i - \mu_x)^2 p_x(x_i)$$

3.

$$\begin{aligned} \phi_x(\omega) &= Ee^{j\omega x} \\ &= \int_{-\infty}^{\infty} e^{j\omega x} f_x(x) dx \end{aligned}$$

where $j = \sqrt{-1}$

4.

$$f_x(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \phi_x(\omega) e^{-j\omega x} d\omega$$



5. $F_{X,Y}(x,y) = P\{X \leq x, Y \leq y\}$

6. $f_{X,Y}(x,y) = \frac{\partial^2}{\partial x \partial y} F_{X,Y}(x,y)$

7. $f_{X,Y}(x,y) = f_X(x)f_Y(y)$

8. $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_{X,Y}(x,y) dy dx = 1$

9. $\phi_{X,Y}(\omega_1, \omega_2) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_{X,Y}(x,y) e^{j\omega_1 x + j\omega_2 y} dy dx$

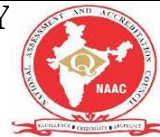
$f_{X,Y}(x,y) = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \phi_{X,Y}(\omega_1, \omega_2) e^{-j\omega_1 x - j\omega_2 y} d\omega_1 d\omega_2$

SHORT ANSWER QUESTIONS:-

- 1. Define expected value of a random variable [Nov-2015]
- 2. Define Joint distribution function with example [Nov-2015]
- 3. Define Joint central Moment [Nov-2015]
- 4. Find the skew for Gaussian distributed random variable [Mar- 2017]
- 5. Distinguish deterministic and non-deterministic processes [Mar-2017]
- 6. Explain the covariance matrix and its properties [Mar-2017]
- 7. Mention the difference between the Variance and Skew. [Mar-2017]
- 8. Explain the equal and unequal distributions. [Mar-2017]
- 9. Write about linear transformations of Gaussian random variables. [Mar-2017]
- 10. Mention the properties covariance . [Mar-2017]
- 11. Write short notes on Chebychev’s inequality. [Apr/May-2018]
- 12. Define Characteristic function and present generation of moments using it. [Apr/May-2018]
- 13 State central limit theorem for the case of equal distributions. [Apr/May-2018]
- 14. Write the properties of jointly Gaussian random variables [Apr/May-2018]

LONG ANSWER QUESTIONS:-

- 1.The random variable X has the discrete variable in the set { -1 , -0.5, 0.7, 1.5, 3} the corresponding probabilities are assumed to be { 0.1 , 0.2, 0.1, 0.4, 0.2} Plot its distribution function and state is it a discrete or continues distribution function. [Nov 2015]
- 2. Discuss Moment generating function and its properties [Nov 2015]
- 3. Calculate E[X] when X is binomially distributed with parameters n and p [Nov 2015]



4. Discuss the probabilities of Joint density function for two random variables X and Y
[Nov-2015]

5 A Joint probability density function is $f(x,y) = 1/ab$ for $0 < x < a$, $0 < y < b$ and $f(x,y) = 0$ elsewhere. Find the joint probability distribution function. **Nov-2015]**

6. Prove that the mean value of a weighted sum of random variables equals the weighted sum of mean values. **[Nov 2015]**

7. Prove that if 'X' 'Y' are two random variables taking real values then $\{E[XY]^2\} \leq E[X^2] \cdot E[Y^2]$.
2015] [Nov

8. What do you mean by probability density function? state and drive its properties **[Mar-2017]**

9. Explain Rayleigh distribution and density functions **Mar-2017]**

10. Find moment generating function about the origin of the poisson distribution **[Mar-2017]**

11. Determine the moment generating function of a random variable with density function $f_x(X) = \frac{1}{b} e^{-(x-a)/b} U(x)$ **[Mar-2017]**

12. Two cards are drawn simultaneously or successively without replacement from a well shuffled pack of 52 cards. Find the mean, variance and standard deviation of the number of kings **[Mar-2017]**

13. State central limit theorem for unequal distributions and explain **[Mar-2017]**

14. Two random variables X_1, X_2 are related to Y as $Y = \sqrt{(X_1^2 + X_2^2)}$. Find the probability density function of y in terms of joint density of X_1, X_2 **[Mar-2017]**

15. Determine the constant b such that the function
 $f_X(x) = 3xy : 0 < x < 1, 0 < y < b$
 $= 0$: other wise **[Mar-2017]**

16. Obtain the relationship between probability and probability density function. **[Mar-2017]**

17. Find the moment generating function of the random variable whose moments are $m_r = (r + 1)! 2^r$. **[Mar-2017]**

18. Write about Chebychev's inequality and mention about its characteristic function. **[Mar-2017]**

19. Determine the moment generating function about origin of the Poisson distribution **[Mar-2017]**

20. Differentiate between the marginal distribution functions, conditional distribution functions and densities. **[Mar-2017]**

21. Given the transformation $y = \cos x$ where x be a uniformly distributed random variable in the interval $(-\pi, \pi)$. Find $f_y(y)$ and $E[y]$. **[Mar-2017]**

22. Let X be a random variable defined, Find $E[3X]$ and $E[X^2]$ given the density function as
 $f_x(x) = \begin{cases} (\pi/16) \cos(\pi x/8), & -4 \leq x \leq 4 \\ 0, & elsewhere \end{cases}$

[Apr/May-2018]



23. Obtain the moment generating function of a uniformly distributed random variable.

[Apr/May-2018]

24. Obtain the variance of Raleigh random variable.

[Apr/May-2018]

25. A random variable X uniformly distributed in the interval $(0, \pi/2)$. Consider the transformation $Y = \sin x$, obtain the pdf of Y.

[Apr/May-2018]

26. Obtain the variance of Gaussian random variable

[Apr/May-2018]

27. The joint characteristic function of two random variables is given by $\phi_{XY}(\omega_1, \omega_2) = \exp(-\omega_1^2 - 4\omega_2^2)$. Check whether X and Y are uncorrelated or not.

[Apr/May-2018]

28. X and Y are statistically independent random variables and $W = X + Y$ obtain the pdf of W.

[Apr/May-2018]

29. Write the properties of joint distribution function.

[Apr/May-2018]

30. Prove that the variance of weighted sum of N random variables equals the weighted sum of all their covariance

[Apr/May-2018]

31. Find the Moment generating function of a uniform random variable distribute over (A, B) and find its first and second moments about origin, from the Moment generating function.

[Nov/dec-2017]

32. A random variable X has a mean of 10 and variance of 9. Find the lower bound on the probability of $(5 < X < 15)$

[Nov/dec-2017]

33. Find the Moment generating function of a random variable X with density function

$$f(x) = \begin{cases} x, & \text{for } 0 \leq x \leq 1 \\ 2 - x, & \text{for } 1 \leq x \leq 2 \\ 0, & \text{else where} \end{cases}$$

[Nov/dec-2017]

34. If X is a Gaussian random variable $N(m, \sigma^2)$, find the density of $Y = PX + Q$, where P and Q are constants

[Nov/dec-2017]

[Nov/dec-2017]

35. If $X_1, X_2, X_3, \dots, X_n$ are 'n' number of independent and Identically distributed random variables, such that $X_k = 1$ with a probability $1/2$; $X_k = -1$ with a probability $1/2$. Find the Characteristic Function of the random Variable $Y = X_1 + X_2 + X_3 + \dots + X_n$.

[Nov/dec-2017]

36. If Independent Random Variables X and Y both of zero mean, have variance 20 and 8 respectively, find the correlation coefficient between the random Variables X+Y and X-Y.

[Nov/dec-2017]

37. Let $X = \cos \theta$ and $Y = \sin \theta$, be two random variables, where θ is also a uniform random variable over $(0, 2\pi)$. Show that X and Y are uncorrelated and not independent.

[Nov/dec-2017]

38. If X is a random variable with mean 3 and variance 2, verify that the random Variables 'X' and $Y = -6X + 22$ are orthogonal

[Nov/dec-2017]

Fill in the Blanks / Choose the Best: (Minimum 10 to 15 with Answers)

1. The probability distribution function $F_X(x)$ of a random variable x then $F_X(\infty) =$ [ans:1]

2. By using _____ density function, the number of defective elements is calculated in a given sample space. [Ans: POISSIONS]



3. How to calculate normalized Gaussian distribution function if mean and variance are given. [ANS: MEAN=0, VARIANCE=1]
4. define uniform probability density if a & b are real constants. [ANS: $f(x)=1/(b-a)$; $a \leq x \leq b$, 0; other wise]
5. In Rayleigh probability density function when the maximum value is occurs _____ [ANS : $df(x)/dx=0$]
6. If X is a discrete random variable and define the expected value of the random variable _____. [ANS: $E[X]=\sum x_i.p(x_i)$]
7. If a random variable X is a constant 'a' what is the expectation of 'a' _____. [ANS: $E[a]=a$]
8. Define nth order moment about origin _____. If the random variable is continuous. [ANS: $m_n = E[x^n] = \int x^n f(x) dx$]
9. The first central moment of 'X' is always equal to _____. [ANS: 0]
10. The skew of the density function $f(x)$ for a random variable is defined as _____ moment. [ANS : THIRD CENTRAL MOMENT]
- 11 Variance of binomial distribution function is [C]
a) n b) np c) npq d) $n^2 pq$
- 12 The second moment about origin is called as [A]
] a) mean squared value b) variance c) standard deviation d) mean value
13. $E[a]=$ _____ where a is arbitrary constant [C]
] a) 0 b) 1 c) a d) ∞
14. It is a measure of a symmetry of the density function of a random variable about its mean [A]
] a) skew b) skewness c) variance d) NONE
15. moments of a random variable are nothing but averages of _____ [C]
] a) mean value b) mean square value c) statistical d) time
16. Variance [constant] value is [D]
] a) -1 b) 1 c) a & b d) 0
17. At what value of w the characteristics function becomes maximum [C]
] a) 1 b) -1 c) 0 d) none
18. If $E[x]=5/12$ Find $E[4x+2]$ [B]
] a) 5/12 b) 11/3 c) 21/5 d) 3/4
19. A coin is tossed 6 times .find probability of getting 5 heads [A]
] a) 2/32 b) 3/64 c) 6/36 d) none
20. If $f(x)=1/6$ for $-3 < x < 3$ & 0 for other wise. Find $P(x < 1)=$ [C]
] a) 2/4 b) 4/2 c) 4/6 d) 6/4



1. $F_{X(t)}(x) = P(X(t) \leq x)$.
2. $f_{X(t)}(x) = \frac{dF_{X(t)}(x)}{dx}$.
3. $R_X(t_1, t_2) = \text{autocorrelation function of the process at times } t_1, t_2 = E(X(t_1)X(t_2))$.
4. $\mu_X(t) = \text{Mean of the random process at } t = E(X(t))$.
5. $R_X(t, t) = EX^2(t) = \text{second moment or mean square value at time } t$.
6. $C_X(t_1, t_2) = E(X(t_1) - \mu_X(t_1))(X(t_2) - \mu_X(t_2))$
 $= R_X(t_1, t_2) - \mu_X(t_1)\mu_X(t_2)$
 $\rho_X(t_1, t_2) = \frac{C_X(t_1, t_2)}{\sqrt{C_X(t_1, t_1)C_X(t_2, t_2)}}$ is called the *correlation coefficient*.
7. The ratio $\rho_X(t_1, t_2)$ is called the *correlation coefficient*.
8. $C_{XY}(t_1, t_2) = E(X(t_1) - \mu_X(t_1))(Y(t_2) - \mu_Y(t_2))$
 $= R_{XY}(t_1, t_2) - \mu_X(t_1)\mu_Y(t_2)$

SHORT ANSWER QUESTIONS:-

1. Write about the following with examples i). discrete time stochastic process ii). Continuous time stochastic process [Nov-2015]
2. Discuss Gaussian random process and state its probability [Apr/May-2018] [Nov-2015]
3. Show that $S_{xx}(\omega) = S_{xx}(-\omega)$. [Nov/dec-2017]
4. Explain WSS and SSS [Nov-2010] [Apr/May-2018]
5. Discuss about Mean and Correlation Ergodic process [Nov-2010]
6. Define autocorrelation function of a random process [Apr/May-2018]
7. Classify random processes and explain [Apr/May 2018]

LONG ANSWER QUESTIONS:

1. Explain the concept of random process and stationary process [Mar-2017]
2. Distinguish between Auto correlation function and cross correlation function. [Mar-2017]
3. State the properties of cross correlation function [Mar-2017]
4. Explain classification of random processes with neat sketch [Mar-2017]
5. State and prove properties of cross correlation function. [Mar-2017]
6. If the PSD of $X(t)$ is $S_{xx}(\omega)$. Find the PSD of $dx(t)/dt$. [Mar-2017]
7. A random process $Y(t) = X(t) - X(t + \tau)$ is defined in terms of a process $X(t)$. That is at least wide sense stationary. a) Show that mean value of $Y(t)$ is 0 even if $X(t)$ has a non Zero mean



value.

[Mar-

2017]

8. If $Y(t) = X(t) + X(t + \tau)$ find $E[Y(t)]$ and $\sigma^2 Y$.

[Mar-2017]

9. Write properties of auto correlation function of a WSS process and prove any three of them.

[Apr/May

2018]

10 A random process $X(t) = A \cos(\omega t) + B \sin(\omega t)$ where ω is a constant and A, B are uncorrelated zero mean random variables with same variances. Check whether X(t) is WSS or not?

[Apr/May 2018]

11. X(t) is a random process with mean =3 and Autocorrelation function $R_{XX}(\tau) = 10 \cdot [\exp(-0.3|\tau|) + 2]$. Find the second central Moment of the random variable $Y = X(3) - X(5)$.

[Apr/May 2018]

12. $X(t) = 2A \cos(Wt + 2\theta)$ is a random Process, where 'θ' is a uniform random variable, over $(0, 2\pi)$. Check the process for mean ergodicity.

[Apr/May 2018]

13. A Random Process $X(t) = A \cdot \cos(2\pi f_c t)$, where A is a Gaussian Random Variable with zero mean and unity variance, is applied to an ideal integrator, that integrates with respect to 't', over $(0, t)$. Check the output of the integrator for stationarity.

[Apr/May 2018]

14. A random Process is defined as $X(t) = 3 \cdot \cos(2\pi t + Y)$, where Y is a random Variable with $p(Y=0) = p(Y=\pi) = 1/2$. Find the mean and Variance of the Random Variable X(2)

[Apr/May 2018]

Fill in the Blanks / Choose the Best: (Minimum 10 to 15 with Answers)

1. In the random process X(s,t) if 's' and 't' are made fixed, then X(s,t) will be a _____.

(number)

2. A random process is said to be stationary, if all its statistical properties do not change with _____.

(time).

3. If a random process is said to be first- order stationary then $f_X(x_1, t_1 + \tau) =$ _____ ($f_X(x_1, t_1)$)

4. If a R.P is said to be first order stationary, then $E[X(t)] =$ _____ (constant).

5. For WSS process. $R_{XX}(t_1, t_1 + \tau) =$ _____ ($R_{XX}(\tau)$)

6. For an ergodic process _____. (time average of the process are equal to the ensemble averages.)



7. Mean value of the random process in true sense _____ . ($\lim_{T \rightarrow \infty} \frac{\int_{-1}^1 X(t) dt}{2T}$)
8. _____ averages are computed by considering all the sample functions. (**ensemble**)
9. All strict sense stationary (SSS) processes are WSS, it is _____. (**true**)
10. For the R.P $X(t) = A \cos \omega t$ where ω is a constant and A is uniform R.V over $(0,1)$, the mean square value is _____ ($\frac{1}{3} \cos(\omega^2 t)$).
11. $R_{XX}(0) =$ _____ **E[x²(t)]**
12. $R_{XX}(\tau)$ is an even function of _____. **(τ)**
13. $|R_{XX}(\tau)| \leq R_{XX}(0)$ it is _____. **(true)**
14. If $x(t)$ is periodic with $2T$, then its ACF is _____. **(periodic with 2T)**
15. If a R.P has a DC component, its ACF function will also have DC component. It is _____. **(true)**
16. $R_{XY}(\tau) =$ _____ **($R_{XY}(-\tau)$)**
17. For jointly WSS process is $C_{XX}(\tau) =$ _____ **($R_{XX}(\tau) - (\bar{x})^2$)**
18. For jointly WSS process, $C_{XY}(\tau) =$ _____ **($R_{XX}(\tau) - \bar{X}\bar{Y}$)**
19. $R_{XX}(a) - \bar{X}^2 =$ _____. **(variance of X)**
20. If $X(t)$ & $Y(t)$ are uncorrelated then _____. **$C_{XY}(t, t+\tau) = 0$**
21. The position R.P $p[X(t), k] =$ For an LTI system, system response $Y(t) =$ _____. **($X(t) \times h(t)$)**
22. Mean value of the system response _____. **($Y(t) = M_x$)**
23. Mean square value of the system response _____ **($\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h(\tau_1) \cdot h(\tau_2) \cdot R_{XX}(\tau_1 - \tau_2) \cdot d\tau_1 d\tau_2$)**
24. Auto correlation function of the system response _____. **($\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h(\tau_1) \cdot h(\tau_2) \cdot R_{XX}(\tau - \tau_2) d\tau_1 \cdot d\tau_2$)**

Unit-IV:

SHORT ANSWER QUESTIONS:-

1. $S_X(\omega) = \int_{-\infty}^{\infty} R_X(\tau) e^{-j\omega\tau} d\tau$

2. $R_X(\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} S_X(\omega) e^{j\omega\tau} d\omega$



the average power of a random process $X(t)$ is

$$EX^2(t) = R_X(0)$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} S_X(\omega) d\omega$$

3.

$$4. S_X(\omega) = \lim_{T \rightarrow \infty} \frac{E|X_T(\omega)|^2}{2T}$$

$$5. S_{XY}(\omega) = \int_{-\infty}^{\infty} R_{XY}(\tau) e^{-j\omega\tau} d\tau$$

$$6. S_{YX}(\omega) = \int_{-\infty}^{\infty} R_{YX}(\tau) e^{-j\omega\tau} d\tau$$

$$7. R_{XY}(\tau) = \int_{-\infty}^{\infty} S_{XY}(\omega) e^{j\omega\tau} d\omega$$

$$8. R_{YX}(\tau) = \int_{-\infty}^{\infty} S_{YX}(\omega) e^{j\omega\tau} d\omega$$

$$9. P_{XY} = \lim_{T \rightarrow \infty} \frac{1}{2T} E \int_{-T}^T X(t)Y(t) dt$$

$$10. P_{XY} = \frac{1}{2\pi} \int_{-\infty}^{\infty} S_{XY}(\omega) d\omega$$

$$11. H(\omega) = FT h(t) = \int_{-\infty}^{\infty} h(t) e^{-j\omega t} dt$$

$$12. S_Y(\omega) = S_X(\omega) H(\omega) H^*(\omega)$$

$$= S_X(\omega) |H(\omega)|^2$$

$$13. R_{XY}(\tau) = h(-\tau) * R_X(\tau)$$

and $R_{YX}(\tau) = h(\tau) * R_X(\tau)$

$$S_{XY}(\omega) = H^*(\omega) S_X(\omega)$$

and

$$S_{YX}(\omega) = H(\omega) S_X(\omega)$$

14.

1. State wiener-khinchin relation.

[Nov 2017] [Mar-2017]



2. Express the relationship between power spectrum and autocorrelation. [Nov 2017]
3. State any 2 properties of the power density spectrum? [Nov 2017]
4. Define cross power spectral density. [Mar-2017]
5. State and explain the relation between power spectrum and Auto-correlation Function. [Mar-2017]
6. Check whether the function below is a valid power density spectrum or not

$$\frac{\omega}{j\omega^6 + \omega^2 + 3}$$

7. Express the relationship between power spectrum and autocorrelation. [Nov/Dec-2016] [Mar-2017]
8. Autocorrelation function of a random process is given by $R_{xx}(T) = 3 \square(T)$. Find and sketch its power density spectrum. [Nov/Dec-2016]
9. Derive the relation between PSDs of input and output random process of an LTI system. [Nov/Dec-2016]
10. Define power spectrum [Nov-2015]
11. Discuss cross power density spectrum [Nov-2015] [Mar-2017]
12. Discuss the spectral characteristic of a system function [Mar-2017]
13. Write the expression for power spectral density. [Apr/May 2018]
14. Write any three properties of cross-power density spectrum [Apr/May 2018]

LONG ANSWER QUESTIONS: -

1. Derive the expression for power density spectrum of a random process. [Mar-2017]
2. Write the properties of power spectral density. [Mar-2017]
3. Find the output PSD and output Auto correlation function for a system with $h(t) = e^{-t}$ for $t > 0$ as input with PSD $h_0/2$ [Mar-2017]
4. Derive the relation between input and output ACF of an LTI system with impulse response $h(t)$. [Mar-2017]
5. Derive the relationship between cross-power spectral density and cross correlation function of a random process. [Nov/Dec-2016]
6. Evaluate the cross power spectral density given the cross correlation of two processes $X(t)$ and $Y(t)$ is $(AB/2)[\sin \omega t + \cos \omega(2t+T)]$, where A, B and ω are constant. [Nov/Dec 2016]
7. Is power density spectrum an even function of ' ω ' or odd function of ' ω '? Justify. [Nov/Dec 2016]
8. Prove $S_{yy}(\omega) = |H(\omega)|^2 S_{xx}(\omega)$. Where $X(t)$ is input process of an LTI system and $Y(t)$ is its output. $|H(\omega)|$ is the transfer function of the LTI system. [Nov/Dec 2016]
9. Define cross power density spectrum and write its properties. [Nov 2015], [Mar-2017]
10. Explain any 4 Properties of Power Density Spectrum. [Dec-2014] [Nov/Dec-2016]
11. Derive the power density spectrum of output of a system, in terms of its input PSD. [Dec-2014]
12. Derive the relationship between Cross PSD & Cross Correlation Function. [Dec-2014]
13. The PSD of random process is given



$$S_{xx}(\omega) = [\pi, |w| < 1 \\ [0, \text{otherwise}]$$

Find its autocorrelation function.

[Dec-2014]

14. Find and plot the auto correlation function of i) wide band White noise ii) band pass white noise

[Mar-2017]

15. Derive the expression for the cross spectral density of the input process $X(t)$ and the output process $Y(t)$ of an LTI system in terms of its transfer function

[Mar-2017]

16. Compare and contrast auto and cross correlation

17. If $Y(t) = A \cos(\omega_0 t + \Theta) + N(t)$, where ' Θ ' is a uniform random variable over $(\pi, -\pi)$ and $N(t)$ is a band limited Gaussian white noise process with $PSD = K/2$. If Θ and $N(t)$ are independent, find the PSD of $Y(t)$

[Nov/Dec-2016]

16. Derive the relationship between cross power spectrum and cross correlation function

[Nov/Dec-2016]

17. The auto correlation function of a random process $R_{XX}(T) = 4 \cos(\omega_0 T)$, where ω_0 is a constant. Obtain its power spectral Density

[Nov/Dec-2016]

18. Obtain the average power in random process $X(t) = A \cos(\omega_0 t + \Theta)$ where A, ω_0 are real constants and Θ is a random variable uniformly distributed in the range $(0, 2\pi)$.

[Mar-2017]

19. Discuss in detail about first order stationary process.

[Nov 2015]

20. The auto correlation function of a random process $X(t)$ is $R_{XX}(\tau) = 3 + s \exp(-4\tau)$. Find the PSD of $X(t)$.

[Nov 2015]

21. Find the PSD of a random process whose autocorrelation function is $R_{XX}(\tau) = A \cos(\omega_0 \tau)$

[Nov

2015]

22. A random process is defined as $Y(t) = X(t) - X(t-a)$ where $S(t)$ is WSS process and $a > 0$ is a constant. Find PSD of $Y(t)$ in terms of the corresponding quantities of $X(t)$.

[Nov

2015]

23. The auto correlation function of a random process $X(t)$ is $R_{XX}(\tau) = 3 + 2 \exp(-4\tau)$.

a) Evaluate the power spectrum and average power of $X(t)$.

b) Calculate the power in the frequency band $-1/\sqrt{2} < \omega < 1/\sqrt{2}$

[Mar-2017]

24. Derive the relation between PSDs of input and output random process of an LTI system

[Mar-2017]

25. Derive the relationship between cross-power spectrum and cross-correlation function.

[Apr/May

2018]

26. The autocorrelation function of a random process $R_{XX}(\tau) = 4 \cos(\omega_0 \tau)$, where ω_0 is a constant. Obtain its power spectral density.

[Apr/May

2018]

27. Obtain the average power in the random process $X(t) = A \cos(\omega_0 t + \square)$ where A, ω_0 are real constants and \square is a random variable uniformly distributed in the range $(0, 2\pi)$.

[Apr/May 2018]



Fill in the Blanks / Choose the Best: (Minimum 10 to 15 with Answers)

1. Parseval's theorem states that (ans: $\int_{-\infty}^{\infty} x^2(t)dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |x(\omega)|^2 dt$.)
2. Average power of a random process $x(t)$ over $(-T, T)$ is
(ans: $P_{xx} = P_{av} = \lim_{n \rightarrow \infty} \frac{1}{2T} \int_{-T}^T E [X^2(T)]$.)
3. The power spectral density is (ans: $S_{xx}(\omega) = \lim_{T \rightarrow \infty} \frac{E[|X_T(\omega)|^2]}{2T}$.)
4. Time average of auto correlation function and PSD forms a (ans: fourier transform) pair.
5. $S_{xx}(\omega) =$ (ans: $\int_{-T}^T R_{xx}(\tau) \cdot e^{-j\omega\tau} d\tau$.)
6. $R_{xx}(\tau) =$ (ans: $\frac{1}{2\pi} \int_{-\infty}^{\infty} S_{xx}(\omega) \cdot e^{-j\omega\tau} d\omega$.)
7. For a W.S.S process, PSD at zero frequency gives _____ $\int_{-\infty}^{\infty} R_{xx}(\tau) d\tau$
8. $\frac{1}{2\pi} \int_{-\infty}^{\infty} S_{xx}(\omega) \cdot d\omega =$ $(R_{xx}(0))$
9. $S_{xx}(\omega)$ is an (ans : even) function of frequency.
10. Fourier transform of $\delta(t)$ is equal to _____ **(1.)**
11. The following which one is wiener-khinchine relation (ans: $R_{xx}(\tau) \leftrightarrow (F.S)S_{xx}(\omega)$.)
12. Cross-power spectral density $S_{xx}(\omega) =$ $\lim_{T \rightarrow \infty} \frac{E[X_T(\omega) \cdot Y_T(\omega)]}{2T}$
13. Cross-power spectral density $S_{yx}(\omega) =$ $\lim_{T \rightarrow \infty} \frac{E[X_T(\omega) \cdot Y_T(\omega)]}{2T}$
14. $S_{xy}(\omega) =$ (ans: both $S_{yx}(-\omega), S_{xy}(\omega)$.)
15. (ans: Real) parts of $S_{xy}(\omega)$ and $S_{yx}(\omega)$ are even functions of frequency ' ω '
16. (ans: Imaginary) part of $S_{xy}(\omega)$ and $S_{yx}(\omega)$ are odd functions of ' frequency ω '.
17. If $x(t)$ and $y(t)$ are uncorrelated and of constant means of $E(x)$ and $E(y)$ respectively, then $S_{XY}(\omega) =$ $(2\pi E(x) \cdot E(y) \cdot \delta(\omega))$
18. CCF and C- PSD forms a (ans: fourier transform) pair.



19. Power spectral density of system response $S_{yy} = |H(\omega)|^2 \cdot S_{xx}(\omega)$.

20. {ans: $S_{xy}(\omega)$ } = $H(\omega) \cdot S_{xx}(\omega)$

21. $S_{yx}(\omega) =$ _____ (ans: $H(-\omega) \cdot S_{xx}(\omega)$.)

22. $S_{xx}(\omega) = \frac{\omega^2}{\omega^6 + 3\omega^2 + 3}$ is _____ (ans: valid PSD.)

23. $S_{xx}(\omega) = \frac{\omega^2}{\omega^4 + 1} - \delta(\omega)$ is valid PSD. This statement is true.

24. A random process has the power density function $S_{xx}(\omega) = \frac{\omega^2}{1 + \omega^2}$. The average power in the process is (ans: $\frac{1}{2}$)

25. The PSD of random process is $S_{xx}(\omega) \begin{cases} \pi; & |\omega| < 1 \\ 0; & \text{elsewhere} \end{cases}$ the ACF is (ans: $\frac{\sin \tau}{\tau}$.)

Unit-V:

SHORT ANSWER QUESTIONS:-

LONG ANSWER QUESTIONS)

Fill in the Blanks / Choose the Best: (Minimum 10 to 15 with Answers)

1. The relation between PSD of flicker noise and frequency $s(w) = 1/f$
2. The thermal noise is also known as **resistor noise and johnson's noise**
3. Thermal noise is given by $\sqrt{4RXTB}$ watts
4. Noise bandwidth $BN = (1/|H(w_0)|^2) \int_{-\infty}^{\infty} |H(w)|^2 dw$ is **true**
5. Mean square value of thermal noise voltage is **4KTB**
6. Thermal noise is **gaussian** in nature
7. Generated voltage and currents is thermal noise are **independent of Frequency**



8. The PSD of thermal noise is $S_i(\omega) = 2KTG/(1-\omega/a)$

9. The constant PSD of white noise is $S_{NN}(\omega) = N_0/2$

10. Extraterrestrial noises are **solar noise and cosmic noise**

11. Available noise power in an electronic ckt is

12. For quadrature component of noise $E[n_c(t) \cdot n_s(t)] = 0$

13. Effective noise temperature $T_e = T_0(1 + \frac{Te}{T_0})$

14. Available noise power spectral density $G_{ao} = G_a(f) \cdot k/2(T_0 + T_e)$ is **true**

15. Total noise power available at the output of the two port network is

$N_{av} = 8a k(T_s + T_e)BN$